INVERSE MAGNETISATION PROBLEM FOR THE ANCIENT ROCKS: A FRUITFUL ENCOUNTER OF HARMONIC ANALYSIS AND PALEOMAGNETISM

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INTRODUCTION / PHYSICAL MOTIVATION

Earth rocks and meteorites may preserve invaluable records of ancient planetary and solar nebula magnetic fields in the form of remanent magnetization. Recent advances in magnetometry have made it possible to measure magnetic fields of very low intensity generated by some rocks, and extraction of this relict magnetic information has become reality in cases that were previously inaccessible using standard rock magnetometers. An endeavor to develop a robust and efficient method for processing these data leads to a number of challenging problems such as effective extension of the restricted measurement data and extraction of certain features of the magnetization (typically, its mean value) that may still allow for retrieving those primordial records without having to solve the entire inverse problem for the underlying spatial distribution of magnetic sources.

In particular, we are concerned with the setup corresponding to a SQUID scanning microscope which operates in a horizontal plane above the sample and measures the vertical component of the magnetic field produced by the sample (a slice of the magnetized rock).

We employ two different solution techniques based on asymptotic analysis and original data continuation approach (involving the use of a priori unknown quantities) which lead to explicit analytical formulas for the magnetic moments in terms of measurement data (see formulas (1)-(2)). One of these techniques is Kelvin transformation followed by asymptotic spherical harmonics projections whereas the second is asymptotic analysis of Fourier integrals in vicinity of the origin. Both methods are novel and lead to essentially identical results which prompts one to explore a deeper connection between elementary ”discrete” and ”continuous” harmonic analysis tools.

MATHEMATICAL FORMULATION

The following setting is a three-dimensional version of what has been previously discussed (see the references in [2]). As follows from Maxwell equation for the stationary field, scalar potential Φ of the magnetic field produced by magnetization $\vec{M}$ (a compactly supported distribution whose support Q is known and corresponds to the area occupied by the magnetic sample) satisfies Poisson equation: $\Delta\Phi = \nabla \cdot \vec{M}$. This establishes a link (in the form of three-dimensional convolution integral) between the unknown magnetization M and the measured quantity $B_3 = -\mu_0 \frac{\partial \Phi}{\partial x_3}$ available on a bounded subset (which we take to be a disk of area A) of a horizontal plane at some height above the sample, μ_0 is a physical constant (magnetic permeability of vacuum). Severe ill-posedness of the reconstruction problem for the whole magnetization distribution and physical interest in overall strength and direction magnetization of a sample lead us to a hope for recovery of the net magnetization vector (essentially, an average magnetization of the sample) $\vec{m} := \iiint_Q \vec{M}(t_1,t_2,t_3)\, d^3t$ which is called magnetic moment.

RESULTS AND DISCUSSION

By using specially developed ”discrete” and ”continuous” harmonic analysis tools mentioned in Introduction and detailed in [1,2], we conclude that the suitable integrals of the measured data provide good approximation to magnetic moment components with the accuracy growing with the size of the measurement area A:

$$m_j = \frac{2}{\mu_0} \iint_{D_A} \left(1 + \frac{4x_j^2}{3A^2} \right) x_j B_3(x_1, x_2)\, dx_1 dx_2 + \mathcal{O}\left(\frac{1}{A^2} \right), \quad j = 1, 2, \tag{1}$$

$$m_3 = \frac{2}{\mu_0 A} \iint_{D_A} B_3(x_1, x_2)\, dx_1 dx_2 + \mathcal{O}\left(\frac{1}{A^2} \right). \tag{2}$$

As shown on Figure 1 obtained for a synthetic example, the method is robust with respect to the 10-percent (i.e. SNR=20 dB) Gaussian white noise once a simple data preprocessing is done. The latter concerns only one component of the magnetization moment whose recovery relies on a necessity dictated by physics implying the vanishing of the total integral of B_3 over the horizontal plane above the sample.

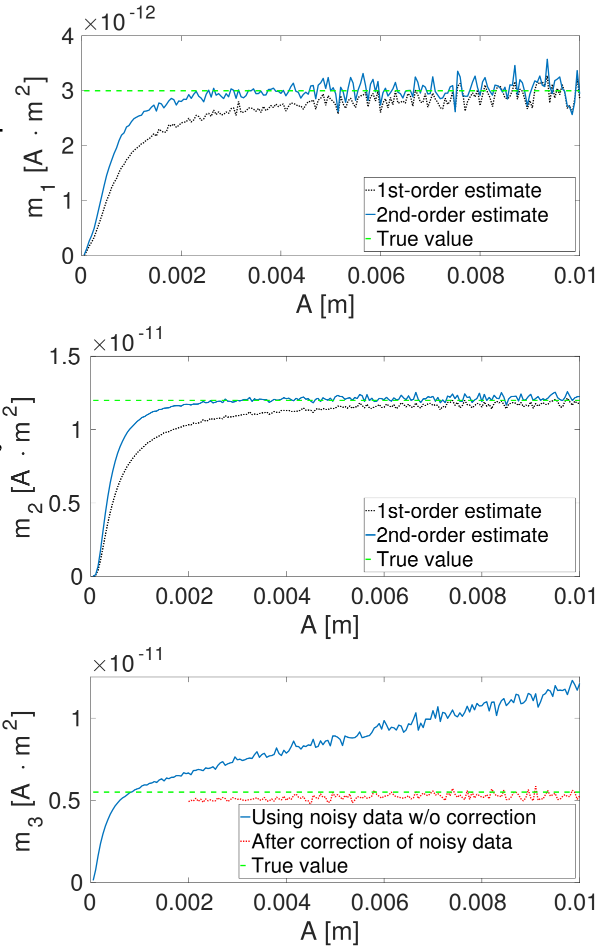

Figure 1: Estimates for components of magnetization moment versus radius A of measurement area

CONCLUSION AND OUTLOOK

We have developed data continuation/analysis techniques which prove to be efficient in a particular inverse magnetization problem (the source enters the governing PDE in the divergence form and the available data is one component of the gradient of the potential field). This naturally raises a question to define a scope of other source-characterization problems (i.e. the triples of source, data and source-related object of interest) where the general philosophy of ”recursive” data continuation and asymptotic harmonic analysis can be used with the same success and at the same time being physically interesting.

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