BUCKLING OF A CLAMPED-CLAMPED BEAM DUE TO INITIAL CURVATURE

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INTRODUCTION

Modern beam theory has a broad range of potential applications. Buckling of beams due to a compressive force is commonly known since Euler. Nowadays, lots of articles are published, where this kind of stability loss is addressed including functionally graded materials and shear effects, often paired with various temperature distributions across the beams cross section [1]. These temperature distributions have similar effect as eigen stresses, which is our chosen approach. Usually, these observation are made in a two-dimensional analysis. As a result to this restriction, these structures become instable only if the resulting normal force is a compressive one. During the validation of finite element schemes for belt drives including natural curvature as imperfections, which are currently developed in our research group, we observed some buckling phenomena even with beams under tension, which we want to study in this work. Hence, in contrast to above, we consider the three-dimensional deformation of the beam, which allows for an additional displacement component as well as torsion. Another interesting and similar problem is the buckling of a flat ring due to initial curvature [2].

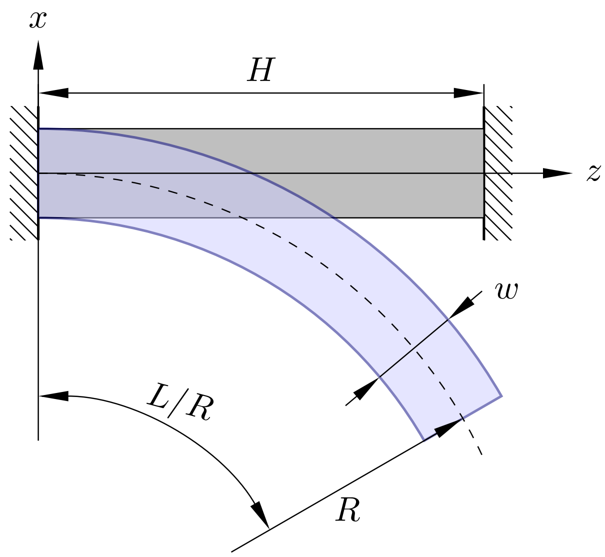

Figure 1: Clamped and free curved beam

PROBLEM STATEMENT AND SOLUTION

We consider a beam with rectangular cross section, which is clamped at both ends. The unclamped beam is curved and its length is unequal to the distance of the clamping conditions, see Fig.1. Due to these initial curvature $\Omega^0 = 1/R$ and pretension ε^0, the straight configuration of the clamped beam is loaded by a normal force as well as a moment. Solving the equations of equilibrium,

$$\mathbf{M}' + \mathbf{r}' \times \mathbf{Q} = 0, \quad \mathbf{Q}' = 0 \qquad (1)$$

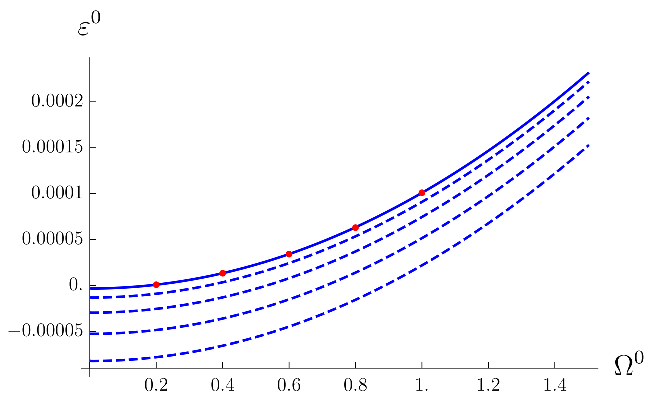

Figure 2: Critical parameters of the clamped beam for $H = 2$, $w = 0.01$ and thickness $h = 0.002$

with moment $\mathbf{M}$, force $\mathbf{Q}$ and position vector $\mathbf{r}$, and their derivatives $()'$ with respect to the material length. We find that the straight configuration is one possible solution. Next, we disturb this equilibrium state and solve the incremental form of the equilibrium equations. An equilibrium solution for a beam without natural curvature but with gravity forces was found and then disturbed by a small initial curvature Ω^0 in the previous study [3]. In contrast to that article, here we obtain a system of homogeneous differential equations for three small rotations, describing the buckling modes of the system. Searching for non-trivial solutions, we find critical combinations of Ω^0 and ε^0. Whereas for vanishing initial curvature we reproduce the Eulerian buckling load, it is also possible to find buckling modes mainly determined by the pre-curvature Ω^0. Further insight will give Ritz-approximations of the corresponding shell problem.

Comparison with finite element solutions of both, beam and shell models show excellent agreement of the critical values of pretension and curvature, see Fig.2. The blue lines are the stability curves for different buckling modes, where the solid line is the critical one. The red dots show points where the stiffness matrix of finite element solutions become singular. It has been observed, that if the beam becomes strip-like, a nonlinear coupling between torsion and tension, altering the torsional rigidity, becomes crucial, which again is in good agreement with shell solutions. The first mode of stability loss of the shell model is presented in Fig.3.

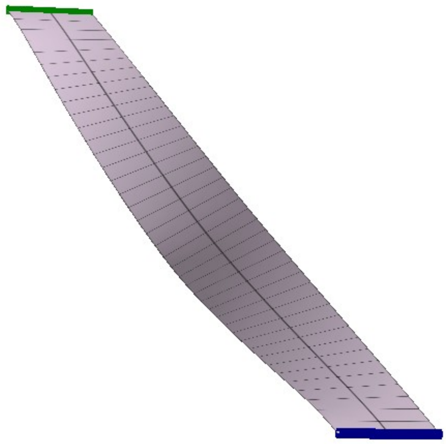

Figure 3: Deformed shell with clamped ends

CONCLUSION

We have seen, that a natural curvature, which is inherent in practical applications like belt drives, may cause interesting buckling phenomena. Although it is interesting from the academical point of view, this study also provided us with a novel benchmark test for the mentioned finite element code.

ACKNOWLEDGEMENTS

The authors thank for the support of the Austrian Research Promotion Agency (FFG), project number 861493.

REFERENCES

[1] Kiani Y., Eslami M.R., *Thermal buckling analysis of functionally graded material beams*, Int J Mech Mater Des, 2010.

[2] Vetyukov Y., *Stability and Supercritical Deformation of a Circular Ring with Intrinsic Curvature*, in Irschik H., Belyaev A., Krommer M, *Dynamics and Control of Advanced Structures and Machines*, p.23-32, Springer (2017)

[3] Vetyukov Y., Schmidrathner C., *A rod model for large bending and torsion of an elastic strip with a geometrical imperfection*, Acta Mech (Accepted).