Experimental Studies of Measurement Uncertainty Relations studied in Neutron Optics

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Neutron interferometry [1], where an interference effect of matter-waves passing through a perfect silicon-crystal interferometer is observed, and neutron polarimetry (also referred to as spin-interferometry) have established as a powerful tool for investigation of fundamental quantum mechanical concepts using massive particles. The former technique enabled several textbook experiments, such as demonstrations of 4π spinor symmetry of spin - 1/2 particles, spin superposition, gravitationally induced phase and topological phase effect, as well as studies of characteristics of intra-partite entanglement, i.e., entanglements between different degrees of freedom. The latter was utilized for demonstration of anti-commuting properties of Pauli spin matrices, topological phase measurements, and tests of alternative theories of quantum mechanics [2].

Heisenberg's uncertainty principle [3] is without any doubt one of the corner stones of modern quantum physics and not only known by scientists but also to the layman. However, the present perception of quantum mechanics has deviated from Heisenberg's empiristic assumptions, reflected in his famous gamma-ray microscope where a measurement process is the source of uncertainty, resulting in a version of the uncertainty relation expressed as a product of widths of probability distributions, i.e., standard deviations (independent of any measurement). These types of uncertainty relations set limits on how sharp the values of two observables can be determined if measured separately, but provide no information of the error when measuring one observable and the thereby induced disturbance on another subsequently (or simultaneously) measured observable. However, a naive product-type error-disturbance uncertainty relation (EDUR) is not valid in general.

Recently, Heisenberg's error-disturbance uncertainty relation has been studied in neutronic and also photonic quantum systems. In 2003, Ozawa thus proposed an improved EDUR, based on rigorous and general theoretical treatments of quantum measurements which is usually refereed to as an *operator-based* approach [4]. In my talk, I am going to give an overview of our neutron optical approaches for investigation of error-disturbance uncertainty relation via a successive measurement of incompatible neutron spin observables (e.g. $\hat{A} = \hat{\sigma}_x$ and $\hat{B} = \hat{\sigma}_y$) [5, 6, 7]. The disturbance $\eta(\hat{B})$ on the observable $\hat{B}$ is induced by the measurement of the observable $\hat{A}$ with error $\varepsilon(\hat{A})$. Though universally valid Ozawa's relations is not optimal. Recently, Branciard [8] has derived a tight EDR, describing the optimal trade-off relation between error $\varepsilon(\hat{A})$ and disturbance $\eta(\hat{B})$. Our experimental results clearly demonstrate the validity of Ozawa's and Branciard's EDRs and that the original Heisenberg EDR is violated throughout a wide range of experimental parameters.

Another more recent experiment tests so called *operational* definitions of error and disturbance developed by Busch and his co-workers. In this theoretical framework error and disturbance are evaluated from the difference between output probability distributions of the successive measurement and reference (ideal) measurements. Despite the ongoing controversy of the two competing approaches, in the case of projectively measured qubit observables, such as non-commuting neutron spin components, both approaches lead to the *same* outcomes [9].

In our recent experiments information-theoretic, or entropic, definitions of error (in this theoretical framework referred to as *noise*) and disturbance are studied. Here, noise and disturbance are defined via correlations between the input states and measurement outcomes. We successfully carried out an experimental test of a newly derived, *tight* noise-disturbance uncertainty relation for general qubit measurements. The noise associated to the measurement of an observable is defined via conditional Shannon entropies and a tradeoff relation between the noises for two arbitrary spin observables is demonstrated [10]. The optimal bound of this tradeoff is experimentally obtained for various non-commuting spin observables. For some of these observables this lower bound can be reached with projective measurements, but we observe that, in other cases, the tradeoff is only saturated by general quantum measurements (i.e. positive-operator valued measures POVMs) as predicted theoretically. These results showcase experimentally the advantage obtainable by general quantum measurements over projective measurements when probing certain uncertainty relations.

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